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μ-Meson Decay into Three Electrons

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The decay $\mu^- \to e^- + e^- + e^+$ via internal conversion is computed using a phenomenological matrix element for the $\mu e\gamma$ interaction. The result is compared with present experimental limits for this process and the results concerning the form factors in the matrix element are discussed. The energy distribution of the emitted electrons is also computed.

IN order to determine whether μ and e differ in any quantum numbers, it is necessary to examine possible decays in which all other particles involved have known quantum numbers. This is not necessarily the case for the normal decay $\mu \to e + \nu_1 + \bar{\nu}_2$, since it is not known *a priori* that ν_2 is the same as ν_1. Examples of transitions which would satisfy these criteria have been given previously.[1] In this note, we investigate the decay $\mu^- \to e^- + e^- + e^+$ by internal conversion using the phenomenological matrix element for $\mu e\gamma$ interactions introduced in reference 1. The decay rate, and the energy distributions of the electrons are computed in terms of the form factors which occur in the phenomenological matrix element, and the implications concerning these form factors, of present limits for the branching ratio $(\mu^- \to 2e^- + e^+)/(\mu \to e + \nu + \bar{\nu})$, are discussed.

The matrix element for $\mu^- \to 2e^- + e^+$ is given by

$$M = ie\left[M_\lambda(e_1)\frac{\bar{u}_+\gamma_\lambda u(e_2)}{q_1^2} - M_\lambda(e_2)\frac{\bar{u}_+\gamma_\lambda u(e_1)}{q_2^2}\right]. \quad (1)$$

Here $M_\lambda(e_1)$ is the quantity given in Eq. (1) of reference 1, which is the $\mu e\gamma$ vertex evaluated on the mass shell of μ and e, and for an electron momentum equal to that of one of the two final state electrons, here called e_1. $M_\lambda(e_2)$ is the corresponding quantity evaluated with electron momentum equal to that of the other final electron. $\bar{u}_+$ is the positron spinor, $u(e_2)$, $u(e_1)$ the electron spinors. $q_1^2 = (p_\mu - p_{e1})^2$ is the momentum transfer when the $\mu e\gamma$ vertex involves e_1, while $q_1^2 = (p_\mu - p_{e2})^2$ is the corresponding quantity when the electron e_2 is emitted at the vertex.

The form of Eq. (1) is implied by the Pauli principle for the 2 final state electrons. The factors $1/q_1^2$ and $1/q_2^2$ are the propagators of virtual photons.

For reference we write the expression for $M_\lambda(e_1)$ here, dropping terms which vanish because $q_\lambda\bar{u}_+\gamma_\lambda u_{1,2} = 0$:

$$M_\lambda(e_1) = \frac{-ie}{(2\pi)^3}\bar{u}(e_1)\left[f_{E0}(q_1^2)\gamma_\lambda + f_{M0}(q_1^2)\gamma_5\gamma_\lambda + \frac{f_{E1}(q_1^2)}{m}\gamma_5\sigma_{\lambda\nu}q_{1\nu} + \frac{f_{M1}(q_1^2)}{m}\sigma_{\lambda\nu}q_{1\nu}\right]u_\mu; \quad (2)$$

m is the muon mass, $\sigma_{\lambda\nu} = (1/2i)[\gamma_\lambda, \gamma_\nu]$.

We assume that each of the form factors, f_{E0}, f_{M0}, f_{E1}, f_{M1} can be expanded in a series in q_1^2. In the case of f_{E0}, f_{M0}, the discussion in reference 1 shows that this expansion must begin with q_1^2, and we will take only this term, so we write

$$f_{E0}(q_1^2) = \bar{f}_{E0}q_1^2,$$
$$f_{M0}(q_1^2) = \bar{f}_{M0}q_1^2. \quad (3)$$

In the case of f_{E1}, f_{M1}, the expansion may contain a term independent of q_1^2, so we keep the first two terms, given

$$f_{E1}(q_1^2) = g_{E1} + \bar{f}_{E1}q_1^2,$$
$$f_{M1}(q_1^2) = g_{M1} + \bar{f}_{M1}q_1^2. \quad (4)$$

The reason for this approximation is as follows. The successive terms in the expansion of the form factors will involve ratios of q_1^2 to some quantity μ^2 with the dimension of mass squared. We may expect that μ^2 will be much larger than the momentum transfer q_1^2 involved in μ decay, since no nonlocal effects have shown up so far in weak interactions at these momentum transfers. It thus appears reasonable to neglect the higher order terms in f_{E0}, f_{M0}. On the other hand, it is known experimentally the g_{E1}, g_{M1}, which appear in the rate for $\mu \to e + \gamma$ are extremely small, or zero, so in f_{E1}, f_{M1} we keep the first two terms.

Using the above equations, it is straightforward to compute the rate $\mu^- \to 2e^- + e^+$. In the computation, we have set $m_e = 0$ everywhere, except in the coefficient of $|g_{E1}|^2$ and $|g_{M1}|^2$. In the latter terms, it is well known[2] that the answer will diverge logarithmically if the electron mass vanishes. In this term, we have therefore not set $m_e = 0$. However, we have dropped some terms of order 1 compared to the term $\ln|m/2m_e|$. This is done in the expectation that $|g_{E1}/m^4|^2 \ll |f_{E0}|^2$, as indicated above.

[1] S. Weinberg and G. Feinberg, Phys. Rev. Letters 3, 111, 244 (1959).

[2] See, e.g., N. M. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955).

We obtain for the decay rate

$$R=2\pi\int\tfrac{1}{2}|M|^2d^3p_1d^3p_2\,\delta(m-E_1-E_2-E_+)$$

$$=\frac{\alpha^2m^5}{16\pi}\Big[|f_{E0}|^2+|\bar{f}_{M0}|^2+\tfrac{3}{10}(|f_{E1}|^2+|\bar{f}_{M1}|^2)+4\,\mathrm{Re}(\bar{f}_{E0}g_{M1}{}^*+\bar{f}_{M0}g_{E1}{}^*)$$

$$+\mathrm{Re}(\bar{f}_{E0}\bar{f}_{M1}{}^*+\bar{f}_{M0}\bar{f}_{E1}{}^*)+\frac{16}{3}\ln\left|\frac{m}{3m_e}\right|\frac{(|g_{E1}|^2+|g_{M1}|^2)}{m^4}+\frac{7}{3}\,\mathrm{Re}(\bar{f}_{E1}g_{E1}{}^*+\bar{f}_{M1}g_{E1}{}^*)\Big].\quad(5)$$

If time reversal invariance holds, it may be shown that all the form factors are relatively real.

It is noteworthy that there is complete symmetry between electric and magnetic form factors in this approximation, when m_e is neglected almost everywhere.

The form we have taken for M_λ would appear to indicate that M_λ might have a singularity for $m=0$. Since this should not be the case in a reasonable theory, it must be that $g_{E1,M1}$ will have a factor like $(m^2)/(\mu^2)$, while $\bar{f}_{E1,M1}$ will contain factors $(m^2)/(\mu^4)$, and $\bar{f}_{E0,M0}$ will have factors $1/\mu^2$. Then since we expect that $m\ll\mu$, the terms in $\bar{f}_{E1}$, $\bar{f}_{M1}$ should contribute very little compared to the $\bar{f}_{E0}$, $\bar{f}_{M0}$ terms.

The size of the coefficient g_{E1}, g_{M1} may be estimated from the decay rate $\mu\rightarrow e+\gamma$ as given in Eq. (3) of reference 1. Using the present limit of 10^{-6} for this rate, we can see that the term

$$\frac{\alpha^2m^5}{16\pi}\left[\frac{16}{3}\ln\left|\frac{m}{2m_e}\right|\frac{(|g_{E1}|^2+|g_{M1}|^2)}{m^4}\right]$$

can give at most a rate for $\mu\rightarrow 3e$ which is 7×10^{-9} the rate for $\mu\rightarrow e+\nu+\bar{\nu}$. Since this is a much smaller rate than the presently known information on the decay $\mu\rightarrow 3e$ we shall neglect both these terms and the interference term between $\bar{f}_{E0}$ and g_{M1}. In this approximation the decay rate becomes

$$R_{3e}=(\alpha^2m^5/16\pi)\{|f_{E0}|^2+|f_{M0}|^2\}$$
$$=10^{-6}m^5\{|f_{E0}|^2+|f_{M0}{}^2|\}.\quad(6)$$

This may be compared with the decay rate for $\mu\rightarrow e+\nu+\bar{\nu}$, given by

$$R_{e\nu\bar{\nu}}=G^2m^5/192\pi^3=1.6\times10^{-4}G^2m^5,$$

where G is the Fermi coupling constant as defined by Feynman and Gell-Mann[3] so that

$$\frac{R_{3e}}{R_{e\nu\bar{\nu}}}=\frac{|f_{E0}|^2+|f_{M0}|^2}{G^2}\times6\times10^{-3}.\quad(7)$$

The present experimental value for this ratio is[4]

$$R_{3e}/R_{e\nu\bar{\nu}}\leqslant10^{-5},$$

so that one obtains

$$(|f_{E0}|^2+|f_{M0}|^2)/G^2\leqslant1.6\times10^{-3}.\quad(8)$$

We have been informed that experiments which would detect the decay $\mu\rightarrow 3e$ at a rate several orders of magnitude less than the limit quoted above are in progress.[5] Such a rate would be comparable to the predictions of a modified intermediate boson theory, with identical ν_1, ν_2.

Finally, we record the distribution of energies of electrons emitted in this decay. The energy spectrum is given by

$$\frac{dN}{d\lambda_1d\lambda_2}=\frac{4m^5\alpha^2}{\pi}\Big\{(|\bar{f}_{E0}|^2+|\bar{f}_{M0}|^2)\left[\frac{13}{2}(\lambda_1+\lambda_2)-2-8\lambda_1\lambda_2-5(\lambda_1{}^2+\lambda_2{}^2)\right]$$

$$+[|\bar{f}_{E1}|^2+|\bar{f}_{M1}|^2][-4\lambda_1{}^2\lambda_2-4\lambda_1\lambda_2{}^2+6\lambda_1\lambda_2+\lambda_1{}^2+\lambda_2{}^2-\tfrac{3}{2}(\lambda_1+\lambda_2)+\tfrac{1}{2}]$$

$$+\mathrm{Re}[\bar{f}_{E0}g_{E1}{}^*+\bar{f}_{M0}g_{E1}{}^*][3(\lambda_1+\lambda_2)-1]+\mathrm{Re}[\bar{f}_{E0}\bar{f}_{M1}+\bar{f}_{M0}\bar{f}_{E1}][4(\lambda_1+\lambda_2)-4(\lambda_1{}^2+\lambda_2{}^2+\lambda_1\lambda_2)-1]$$

$$+[|g_{E1}|^2+|g_{M1}|^2]\left[\frac{2\lambda_1\lambda_2+2\lambda_2{}^2-2\lambda_2-\frac{1}{2}\lambda_1+\frac{1}{2}}{1-2\lambda_1+(m_e{}^2/m^2)}+\frac{2\lambda_1\lambda_2+2\lambda_1{}^2-2\lambda_1-\frac{1}{2}\lambda_2+\frac{1}{2}}{1-2\lambda_2+(m_e{}^2/m^2)}+\text{constant}\right]$$

$$+\mathrm{Re}[\bar{f}_{E1}g_{E1}{}^*+\bar{f}_{M1}g_{M1}{}^*][2(\lambda_1{}^2+\lambda_2{}^2)+4\lambda_1\lambda_2-2(\lambda_1+\lambda_2)+1]\Big\}.$$

Here λ_1, λ_2 are the two electron energies in units of the muon mass. The notation (+ constant) in the coefficient of $|g_{E1}|^2+|g_{M1}|^2$ indicates that we have neglected some terms which are order 1 when $\lambda_1\rightarrow\frac{1}{2}$, compared to terms retained, which go to $(m^2)/(m_e{}^2)$.

The terms other than the one involving $|g_{E1}|^2+|g_{M1}|^2$ are slowly varying functions of energy, with maximum in the neighborhood of $\lambda_1=\frac{1}{3}$, which corresponds to the particles emitted at angles of 120°.

[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[4] J. Lee and N. P. Samios, Phys. Rev. Letters **3**, 55 (1958).
[5] S. Penman (private communication).

The term with $|g_{E1}|^2+|g_{M1}|^2$ is very large for λ_1 or λ_2 at or near $\frac{1}{2}$, which corresponds to one of the electrons coming out in the same direction are the positron. This behavior of the energy spectrum of pairs coming from internal conversion when the real photon decay is allowed is also well known.[2] However, in view of the fact that $|g_{E1}|^2+|g_{M1}|^2$ is very small or zero, this term is probably unimportant.

We conclude that present information about $\mu\rightarrow 3e$ does not give such sensitive restrictions on the $\mu e\gamma$ form factors as other measurements, such as $\mu+p\rightarrow e+p$. However, they are consistent with the vanishing of all such form factors, and future searches for $\mu\rightarrow 3e$ may be sensitive enough to lower the limits on the form factors to those values predicted by the intermediate boson theory.

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Pion-Pion Interactions in τ and τ' Decays*

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The final-state interactions in τ and τ' decays of K^+ mesons are studied by means of a Mandelstam representation. It is assumed that only the S-wave pion-pion scattering amplitude is large enough to have an appreciable imaginary part. Coupled, linear, singular, integral equations are found for the amplitudes describing $K^++\pi\rightarrow\pi+\pi$ in the physical and unphysical regions. From the solutions to these equations the τ and τ' decay matrix elements may be constructed. In an approximation the equations are solved in terms of pion-pion phase shifts. Comparison with experiment is made using Chew and Mandelstam's solution of the corresponding nonlinear equations for $\pi-\pi$ scattering. Consistency with experiment is found for values of the coupling constant implying repulsive $T=0$ and $T=2$ phase shifts. Independently of the approximation the equations show that the implications in τ and τ' decay of the rule, $\Delta T=\frac{1}{2}$, $\frac{3}{2}$, are identical with those of the $\Delta T=\frac{1}{2}$ rule. Hence the energy spectra in τ' decay cannot be a critical test of the $\Delta T=\frac{1}{2}$ rule. Also, independently of the approximation, it is shown that the decay matrix element cannot be expanded in a series of integral powers of the pion kinetic energies.

1. INTRODUCTION

IN a recent analysis of 900 τ^+ decays of K^+ mesons, $K^+\rightarrow\pi^++\pi^++\pi^-$, it was found that the π^- energy spectrum differs significantly from that predicted from the density of states alone.[1] The minimum pion wavelength in this process is about one pion Compton wavelength. If the weak interaction proceeds in some way through the heavy fermion pairs and has a range determined by the intermediate mass, one would expect very little dependence upon the pion momenta in the matrix element, because the pion wavelength is large compared to the radius of interaction. Hence the observed deviation from a constant matrix element is some evidence for final-state interactions which extend the spatial region of interaction for the outgoing pions. This is an attractive possibility in view of the current interest in pion-pion interactions. The system of three low-energy pions should be an ideal analyzer for such effects.

Thomas and Holladay[2] have investigated the effect of an attractive, $T=2$, pion-pion interaction in τ decay by using Watson's final-state interaction formalism.[3] This method is suited to discussion of the final-state scattering of a single pair of particles. It is applicable to the problem of τ^+ decay with a $T=2$, $\pi-\pi$ force because the state of two π^+ is pure $T=2$ and the other pairs, $\pi^+\pi^-$, are predominantly $T=0$ in the S wave. If, however, there are strong $T=0$ pion-pion effects, then a formulation is required which is capable of dealing simultaneously with interactions between various pairs. The Mandelstam representation can serve as the basis for such an analysis, just as an ordinary single variable dispersion relation can be used to discuss a single final-state interaction.

In the present work Mandelstam representations are assumed for the τ and τ' decay amplitudes.[3a] It is assumed that only the S-wave pion-pion scattering amplitudes are large (have an imaginary part). Linear integral equations are obtained for the $\tau+\pi\rightarrow\pi+\pi$ scattering amplitudes in the physical and unphysical regions. These equations involve the pion-pion S-wave phase shifts. From the solutions the Mandelstam representations for τ and τ' decays may be constructed. In general, two new parameters are needed to characterize τ decay. The $\Delta T=\frac{1}{2}$, $\frac{3}{2}$ rule can be applied to yield a simpler set of equations with only one parameter, which, if solved,

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[1] S. McKenna, S. Natali, M. O'Connell, J. Tietge, and N. C. Varshneya, Nuovo cimento **10**, 763 (1958).

[2] B. S. Thomas and W. G. Holladay, Phys. Rev. **115**, 1329 (1958).

[3] Also, A. N. Mitra has investigated the effects of a $T=2$ pion-pion resonance in the τ decay [Nuclear Phys. **6**, 404 (1958)].

[3a] N. N. Khuri and S. B. Treiman have also considered this problem (Phys. Rev. to be published).